Measuring the train timetables robustness

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Abstract
Generating timetables for train movements such that the total travel time of trains are minimized is studied more that 35 years in the literature. Up to now, many different methods same as exact, numeration, heuristics, simulation and knowledge-based ones have been presented. It can be easily observed that generally occurring disruptions in train movements are experienced in practice which results in non valid timetables. In these situations a real-time scheduling can be used which in fact is accompanied with some other difficulties. Based on the above fact finding timetables which have the ability of absorbing a pre-defined range of interruptions are highly invaluable. In this paper after a brief review about the robust timetables, two new methods are proposed to measure the robustness of train schedules. Finally an interesting example is presented to validate the introduced indices.

Keywords: Robust time tables, Train scheduling, measuring index

1. Introduction

One of the most important pillars of a modern economy is the ability to transport goods and people. Economy is substantially benefited from convenient transportation facilities accompanied with a comprehensive integrated planning. Railway industries are known as the safest one among any other existing facilities. The primary role of railway is to transport passengers and freights. In this regard the level of service granted to the customers, either passengers or owner's freights is a key factor to attract more willing in using the rail other than different transportation alternatives. It is clear that rail transport planning fundamentally influences customer's satisfaction. This planning consists of several steps: Analyzing passenger's demand, line planning, train schedule planning, rolling stock planning and crew management [1].

Train scheduling is considered as the most challenging problems in railway planning which affects the interests of customers and the cost saving utilization of existed infrastructures. A train timetable defines the planed arrival and departure times of trains to/from stations. The classical objective function is to minimize the delays of trains to their destinations. The idea could also be extended when other kinds of objectives like the minimization of the deviation from the working hours of crews as well as the fuel consumptions, are considered. On the other hand, there are many practical cases where, a small change in data could make the optimal solution virtually infeasible. The primary objective of this paper is to present an algorithm to measure the robustness of a timetable.

Carey [1], presents some heuristic measures of stability of train timetables in two different categories: using and not using probabilities. In the paper some reasons are counted not to use the past information, probability of disruptions, in making robustness measures:

1- We would ideally need separate data on exogenous delays and knock-on delays, but in the available data these may not be separable.
2- Due to changes in the causes of delays, predicting the future delays may differ from past exogenous delays.
3- An ideal tool to make robust timetables is to increase the headways which indeed do not deal with the probabilities.

Two rules make the backbones of introduced heuristic measures of the second category:

1- Making headways more equal for all trains of the same type, reduces the probability of knock-on delays.
2- To improve the reliability we should make the smallest headways as large as possible.

In [2], Salido, et al, introduces two indices for measuring the robustness of timetables. To evaluate the robustness approaches, some real timetables from the Spanish railway infrastructure is used.

Vroman, et al, [3] explain that an effective way to increase the reliability of timetables is to reduce the propagation of delays generated by the correlations and mutual relations of trains and also heterogeneity of trains. The introduced method is relied on the mean headway in addition to the speed of trains.

Hoogheimstra and Teunisse [4], Hofman and Madsen [5] use simulation methods to measure the robustness of timetables. Some other related researches have been conducted by Goverde and Odijk [6] and Odijk, et al [7].

The paper is organized as follows. First In section 2, an overview to robustness is presented. In section 3, two measurement analytical methods, are introduced and are validated by studying an interesting example. The conclusion remarks are given at the end to summarize the contribution of this paper.

2. An overview to robustness

A disruption, depends on its size, may affect only one train, or can be propagated through other trains. To clarify the issue, consider figure 1. Suppose that an interruption leads train *i* to arrive *t´* time units later than expected to station *h*. In this situation if *t*> *t´* then the noise do not influence train *j*, but if *t*<*t´* then the interruption will cause some delays for train j as well.

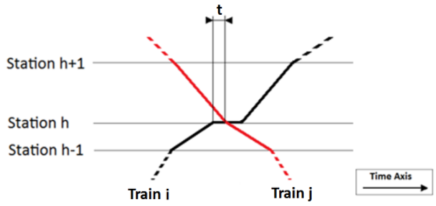

Figure 1: Secondary delay

Practically running of trains are always accompanied with many interruptions and disturbances. The type and size of the interruptions are varied from just some seconds to even some hours. For example mounting and dismounting of passengers is always affected by minor noises, on the other hand track damage may need hours to be managed. Timetables can be divided into two categories: first a primary timetable which is generated before running of trains and second real-time timetables which must be generated during operations after occurrences of some disturbances. In this paper we focus on the primary timetables. An ideal primary timetable is the one that not only minimizes all related costs but also offer robustness against minor disruptions. There is a trade-off among optimality, capacity and robustness. Figure 2 depicts that as greater capacity is used; greater will be the effects of noises to the time tables.

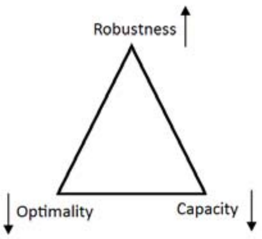

Figure 2: trade-off among optimality, capacity and robustness

Different definitions of robustness are presented in the literature. Followings are the ones which are a function of time:

Definition 1: t-robustness of timetable x, R(x,t) is defined as the percentage of disruptions lower than *t* time units that the timetable is able to tolerate without any modifications in traffic operations (crossing, overtaken, etc). [2]

Definition 2: a timetable is *(t,k)-robust* if upon an disruption lower than *t* time units, it is able to return to the initial stage after *k* time units. [9]

We have introduced following definition.

Definition 3: t-robustness of timetable x, R(x,t) is defined as the percentage of absorbed disruptions up to *t* time units by the timetable without any modifications in traffic operations.

To clarify the third definition of robustness, consider timetable *x*. *t* time units disruption can be induced in any parts of the timetable. This disturbance, depending on its size, can only affect one train or may be propagated through the timetable.

Mainly two factors affect the robustness of a timetable:

1- The amount of buffer times between each two events, i.e. arrival and departures of trains.

2- The amount of time supplements added to the travelling time of trains.

In a single-track railway line, the first factor is more effective comparing the second one. The reason relates to the fact that it directly deals with minimizing the propagation of secondary delays.

3. The measurement analytical methods

In this paper we have proposed two different methods to measure the amount of robustness of timetables. Both of them concentrate only on the amount of buffer times. Simply more buffer times result in more robustness. On the other hand the place and the size of buffer times are highly important.

Method 1:

Disruptions are based on different probability distributions. Generally three assumptions are considered concerning the distributions:

1- The interruptions obey an unknown but symmetric distribution in $[-\mathrm{a}, \mathrm{a}]$.
2- The interruptions obey a symmetric distribution in $[-\mathrm{a}, \mathrm{a}]$, where P(t)>P(t´) ,$\forall \mathrm{t}, \mathrm{t}' \in [-\mathrm{a}, \mathrm{a}], t < t'$, e.g. a Triangular Distribution.
3- The interruptions obey a uniform distribution in $[-\mathrm{a}, \mathrm{a}]$, i.e. *P*(*t*)=*P*(*t´*) ,$\forall \mathrm{t}, \mathrm{t}' \in [-\mathrm{a}, \mathrm{a}]$

Note that, *P*(*t*) refers to the probability of occurrence of a t-disruption in a timetable.

Now suppose that the probability distribution of interruptions is known, in this case we have assigned a value to each buffer time depending on the cumulative probability of the occurrence of equal interruptions. In other words *V*(*t*), i.e. the value of a t-buffer time, equals to $V(b) = \sum_{b' \le b} p(b')$; *or* $V(b) = \int_{b'=0}^{b} f(b')$, for discrete or continuous distributions.

In addition to the value of the buffer times, each buffer time has different effect upon the amount of robustness of timetables. To clarify the issue consider figure 3. We intend to measure the effects of 4 different buffers.

- Buffer no.1 has effect on trains 1 and 3 directly and on trains 2 and 3 indirectly.
- Buffer no.2 has effect on trains 1 and 4 directly and on train 2 indirectly.
- Buffer no.3 has effect on trains 2 and 3 directly and on train 4 indirectly.
- Buffer no.4 has just effect on trains 2 and 4 directly.

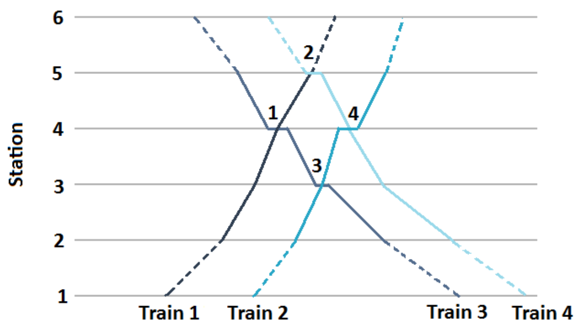

Figure 3: Different buffer times

In this example buffer no. 1 has more effect on the robustness of the timetable comparing the others. Finally the proposed index for measuring the robustness of timetable *x*, is as follows:

$R(x) = \sum_b [V(b) \times V'(b,x)]$

where,

D(*b*) refers to the number of trains which are directly affected by buffer time *b*.

ID(b) refers to the number of trains which are indirectly affected by buffer time *b*.

α indicates the proportion of indirect to direct effects, 0<*α*<1.

Method 2:

Salido et al, [3] proposed equation 1 to measure the robustness of a time table.

$$R(x,t) = 100 \times \frac{NAbsorobed\ Disruptions(x,t)}{T \times S} \qquad (1)$$

Where, *NAbsorbedDisruptions* (*x*,*t*) studies all crossings and returns the number of disruptions that can be absorbed with the available buffer times. *T* and *S* refer to the number of trains and stations, respectively.

The weakness of this index is that by the assumption of a big disruption, the introduced index for all timetables returns 0. Moreover the index does not consider the possible effects of disruptions to other trains. To that end we have developed the formula as shown in equation 2.

$$R(x,t) = 100 \times \frac{|B| \times |T| \times t - \sum_{i \in T} \sum_{b \in B} NAD_{i,b}^{x,t}}{|B| \times |T| \times t} \qquad (2)$$

where, $NAD_{i,b}^{x,t}$ stands for *Non Absorbed Delays* and refers to the amount of delays which are not absorbed by timetable *x*, when train *i* is affected by a *t*-disruption in block section *b*.

Before presenting the algorithm which describes the procedure to find the $NAD_{i,s}^{x,t}$, consider figure 4 as a part of a timeable. Figure 5 depicts the situation in which train 1 is disturbed in the block section between *v* and *w*.

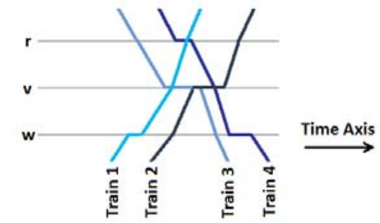

Figure 4: Primary timetable

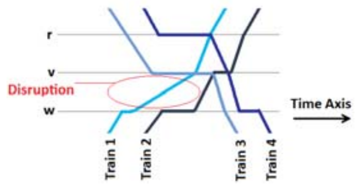

Figure 5: Under perturbation timetable

Figure 5 shows the propagation of delays among other trains.

The proposed algorithm must consider all the alterations occurred in the whole timetable.

Table 1: Used notations in algorithm 1

Symbol	Definition
T	Set of trains
B	Set of Block sections
L_b	List of perturbed trains in block section *b*
Dep(i,b)	Primary departure time of train *i* from the beginning of block section *b*
Arr(i,b)	Primary arrival time of train *i* to the end of block section *b*
DepN(i,b)	Real-time departure time of train *i* from the beginning of block section *b*
ArrN(i,b)	Real-time arrival time of train *i* to the end of block section *b*
E_i	Origin of train *i*
dn	Disruption induced to the timetable
b^+	Block section increasing counter
b^-	Block section decreasing counter
Next(j,b)	The block section which is passed by train *j* after block section *b*
*t(p,b)**	The number of train which has priority *p* in passing block section *b*

* As a train passes a block section sooner in the primary time table, the allocated priority will be larger.

Algorithm 1: Total amount of non-absorbed delays in the case of *dn*-disruption for train *i* in block section *b´*

```
b=b´
b⁺=b⁻=b´
Lb ← i
While Lb is not empty do
    ArrN(i,b)=Arr(i,b)+dn
    For j∈T do
        If (Dep(j,b)>Arr(i,b)) and (Dep(j,b)<ArrN(i,b)) then
            Lb←j
            DepN(j,b)=ArrN(i,b)
            ArrN(j,b)=Arr(j,b)+[DepN(j,b)-Dep(j,b)]
        end
    end
    For j∈Lb do
        For p=1 to |T| × |B|
            For p´= 1 to p-1 do
                Update (ArrN, DepN)
            end
        end
    end
    For j∈Lb do
        If b<>Ej then
            Lnext(j,b) ← i
        end
    end
    Do
        b⁺=b⁺+1
        b=b⁺
    Or
        b⁻=b⁻+1
        b=b⁻
end
Print Output
```

Function: Update (ArrN, DepN)

```
k= t(p´,b)
If DepN(j,b)<ArrN(k,b) then
    ArrN(j,b)=Arr(j,b)+[ArrN(k,b)-DepN(j,b)]
    DepN(j,b)=ArrN(k,b)
end
```

Function: Output

```
For b∈B do
    For j∈T do
        If b=Ej then
            If j∈Lb then NAD=NAD+(ArrN(j,b)-Arr(j,b))
        end
    end
end
return NAD
```

Experimental example

To evaluate the propsed methods, two different timtables as shown in figure 6 and 7 are studied. The input parameters for both time tables are the same.

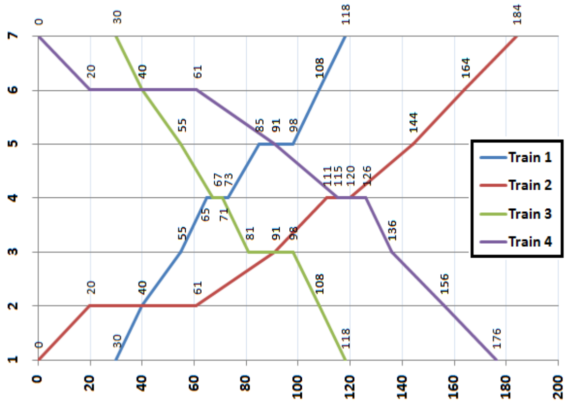

Figure 6: First Timetable

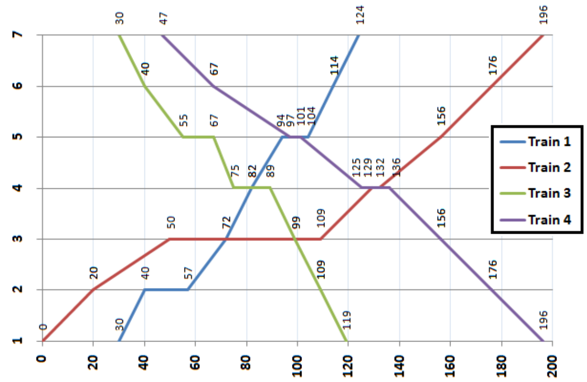

Figure 7: Second Timetable

The details of the first method are shown in table 2. Note that the buffers are numbered from left to right, e.g. in the second timetable buffer no. 1 is referred to the stop of train no.2 in station 2. Based on the results of table 2, disregarding the amount of parameter α and also the probability of disruptions, timetable 1 is more robust that the second one.

Table 2: Details of first index computation

Buffer No.	Direct effect (1)	Indirect effect (1)	Direct effect (2)	Indirect effect (2)
1	2	2	1	1
2	2	2	1	1
3	2	2	2	2
4	2	1	2	1
5	2	1	3	2
6	2	0	2	0
Total	**12**	**8**	**11**	**7**

Table 3, outline the details of results of the second index. By using the second method, we arrived at 0.494 and 0.283 for timetables 1 and 2, respectively.

Table 3: Details of second index computation

Train	Block	Total Delay(1)	Total Delay(2)	Train	Block	Total Delay(1)	Total Delay(2)
1	1	0	0	3	1	15	15
1	2	0	14	3	2	15	15
1	3	0	14	3	3	0	17
1	4	2	6	3	4	0	1
1	5	15	15	3	5	0	0
1	6	15	15	3	6	0	0
2	1	0	0	4	1	15	15
2	2	8	0	4	2	15	15

2	3	8	20	4	3	15	15
2	4	15	15	4	4	6	12
2	5	15	15	4	5	8	12
2	6	15	15	4	6	0	12

Therefore based on the found results, Both presented methods confirmed that the timetable 1 is more robust than the second one.

Conclusion

In this paper firstly a brief review about the definition and characteristics of robust timetables has been introduced, and then two new methods have been proposed to measure the robustness of train schedules. Finally an interesting example was presented to validate the introduced indices. The results approved the efficiency of both methods.

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